Maple Demonstration - Year 13

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Introduction to Maple

Before we start, let's make sure Maple is ready to go. Restart means all variables are cleared and we are starting 'fresh'. We have to enter "math mode" (where your cursor is surrounded by a dashed box) by pressing F5, and we use a semi-colon to finish the command. We then press enter to run the command:
restart;

What is Maple? To start, it's a calculator: $1 + 1 =$. We press ctrl+= to compute something inline (the answer appears in blue), or press enter to get the output on a new line.
$2^{1000} - 1$;

```
107150860718626732094842504906000181056140481170553360744375038837035105112493\
612249319837881569585812759467291755314682518714528569231404359845775746985\
748039345677748242309854210746050623711418779541821530464749835819412673987\
675591655439460770629145711964776865421676604298316526243868372056680693 75
```

(1)

But it can do a lot more than a calculator! Maple treats everything symbolically so it can deal with more than just numbers. It can do algebra:
$expand\left((x+1)^3 \cdot (x-1)^3 - (x-2)^2 \right)$;

$$x^6 - 3\,x^4 + 2\,x^2 + 4\,x - 5 \qquad \textbf{(2)}$$

$factor\left(x^{10} - 1\right)$;

$$(x-1)\,(x+1)\,\left(x^4 + x^3 + x^2 + x + 1\right)\,\left(x^4 - x^3 + x^2 - x + 1\right) \qquad \textbf{(3)}$$

We can also solve equations, both with exact solutions and in terms of other variables:
$solve\left(x^4 - 10\,x^3 + 35\,x^2 - 50\,x + 24\right)$;

$$1, 2, 3, 4 \qquad \textbf{(4)}$$

$solve\left(a \cdot x^2 + b \cdot x + c, x\right)$;

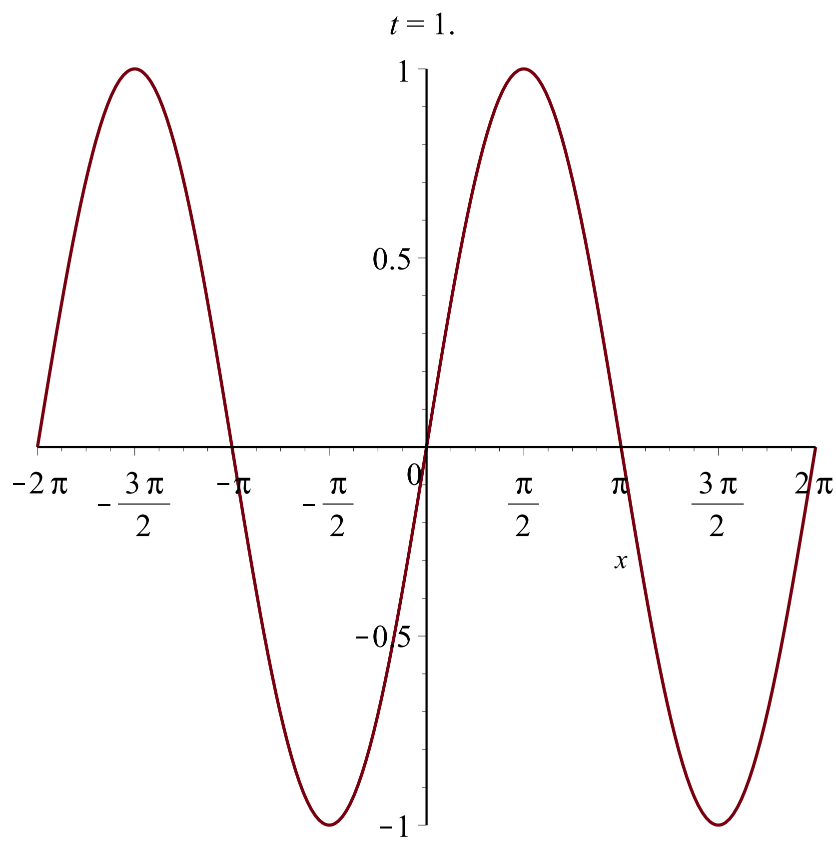
$$\frac{1}{2}\,\frac{-b + \sqrt{-4\,a\,c + b^2}}{a}, -\frac{1}{2}\,\frac{b + \sqrt{-4\,a\,c + b^2}}{a} \qquad \textbf{(5)}$$

Be careful though! Maple will solve **exactly** what you ask it to. For example, if you write ab and not a*b, it will think ab is a new variable!
$is(ab = a \cdot b)$;

$$false \qquad \textbf{(6)}$$

$solve\left(ax^2 + bx + c, x\right)$;
The lack of output means Maple cannot find any solutions - mainly because Maple thinks x doesn't occur as a variable.

Maple is also really useful for visualising mathematics. It can do cool things like animated graphs to see how a parameter changes a function:
$plots\text{:-}animate\left(plot, \left[\sin(x \cdot t), x = -2 \cdot \mathrm{Pi} .. 2 \cdot \mathrm{Pi}\right], t = 1 .. 10\right)$;



More on that later.

To define a variable we use the := symbol
$x := 5;$

$$5 \tag{7}$$

And we can check what the value of x is:
$x;$

$$5 \tag{8}$$

Careful! Now x will be 5 unless you undefine it:
$f := x^3 + \mathrm{x} - 7;$

$$123 \tag{9}$$

So we have to redefine x to 'reset' it
$x :=' x';$

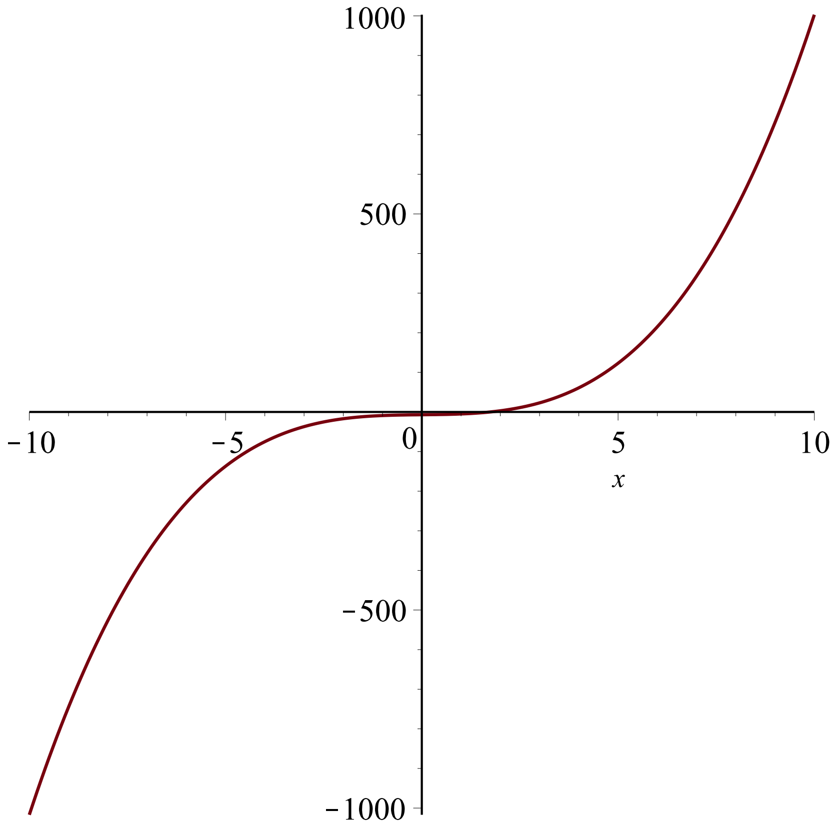
$$x \tag{10}$$

And now we can define f as we want
$f := x^3 + \mathrm{x} - 7;$

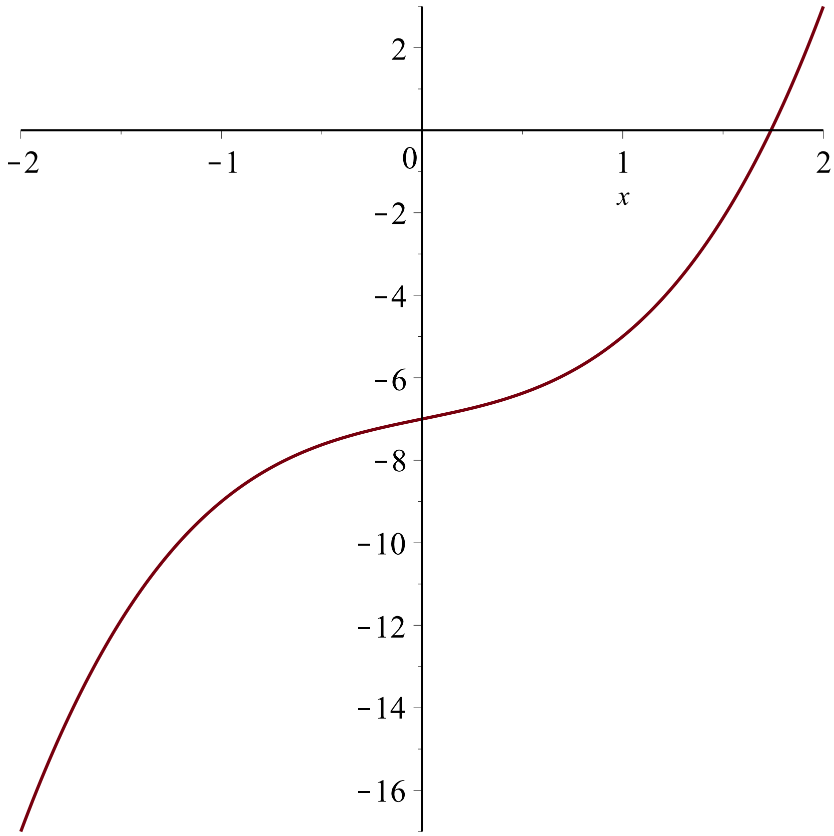
$$x^3 + x - 7 \tag{11}$$

Notice that we have had a semi-colon at the end of each line. If we use a full colon then the command runs, but we don't see any output:

$f := x^3 + x - 7 :$
Now let us see what f looks like
$plot(f);$



This is a bit of a wide zoom - we can't see where it intersects the x-axis. Let's check by specifying what we want to see.
$plot(f, x=-2..2)$



We can also click on the zoom tool to zoom in by dragging an area, or using the scroll button on the mouse.

We can do a lot more with a function than just plot it or solve it. We can differentiate it:
diff (*f*, *x*);

$$3x^2 + 1 \quad \textbf{(12)}$$

Or we can integrate it.
int(*f*, *x*);

$$\frac{1}{4}x^4 + \frac{1}{2}x^2 - 7x \quad \textbf{(13)}$$

$intvalue := int\left(f, x = \frac{\mathrm{sqrt}(2)}{3} ..5 \cdot \mathrm{Pi}\right);$

$$\frac{625}{4}\pi^4 - \frac{10}{81} + \frac{25}{2}\pi^2 - 35\pi + \frac{7}{3}\sqrt{2} \quad \textbf{(14)}$$

Notice that Maple gave us an exact answer. It will always try to give an exact answer when it can. We can see the value of this by using the evalf command.
evalf (*intvalue*)

$$15236.76117 \quad \textbf{(15)}$$

And we can get a numerical answer to as many digits as we want:

$evalf(\text{Pi}, 1000)$

3.14159265358979323846264338327950288419716939937510582097494459230781640628620\
899862803482534211706798214808651328230664709384460955058223172535940812848\
111745028410270193852110555964462294895493038196442881097566593344612847564\
823378678316527120190914564856692346034861045432664821339360726024914127372\
458700660631558817488152092096282925409171536436789259036001133053054882046\
652138414695194151160943305727036575959195309218611738193261179310511854807\
446237996274956735188575272489122793818301194912983367336244065664308602139\
494639522473719070217986094370277053921717629317675238467481846766940513200\
056812714526356082778577134275778960917363717872146844090122495343014654958\
537105079227968925892354201995611212902196086403441815981362977477130996051\
870721134999999837297804995105973173281609631859502445945534690830264252230\
825334468503526193118817101000313783875288658753320838142061717766914730359\
825349042875546873115956286388235378759375195778185778053217122680661300192\
787661119590921642019 9 **(16)**

There are lots of useful assistants for Maple which can be found under the Tools option in the menu bar. Assistants help you create things in Maple (plots, equations etc), Math Apps show you different concepts, Tutors help you understand how to do certain things in Maple, and Tasks are templates to do certain things.

The best place to find help is **Maple Help.** You can find it under Help in the toolbar.

Some 'Programming'

You can also 'program' with Maple. For example we can have a look at the first 100 Fibonacci numbers. You can create a list using [] and access elements by using [] too. We fill it with a 'for' loop:

```
FibNums := [0$100] :
FibNums[1] := 1 :
 FibNums[2] := 1 :
for i from 3 to 100 do
  FibNums[i] := FibNums[i − 2] + FibNums[i − 1] :
 end do:
```

This has filled the list with the Fibonacci numbers and we can look at the final element to see the 100th Fibonacci Number

$FibNums[100]$

$$354224848179261915075 \quad \textbf{(17)}$$

We can also create 'procedures' which are functions to do things in Maple. If you are doing something over and over then creating a procedure can be helpful.

```
myProc := proc(x)
  local i, runningtotal :
  runningtotal := 0 :
  for i from 1 to 10 do:
    runningtotal := runningtotal·x + x :
  end do:
  return runningtotal :
 end proc:
myProc(9)
```

$$3922632450 \tag{18}$$

Modular Arithmetic

To calculate a number mod p in Maple, just write mod:

$(9 \cdot 12 + 18)$ **mod** 11

$$5 \tag{19}$$

What if we wanted to solve 7x+2 ≡1 (mod 31)? We could try every value...

```
for i from 0 to 30 do
  print(i, " evaluates to", (7·i + 2) mod 31) :
 end do:
```

0, " evaluates to", 2

1, " evaluates to", 9

2, " evaluates to", 16

3, " evaluates to", 23

4, " evaluates to", 30

5, " evaluates to", 6

6, " evaluates to", 13

7, " evaluates to", 20

8, " evaluates to", 27

9, " evaluates to", 3

10, " evaluates to", 10

11, " evaluates to", 17

12, " evaluates to", 24

13, " evaluates to", 0

14, " evaluates to", 7

15, " evaluates to", 14

16, " evaluates to", 21

17, " evaluates to", 28

18, " evaluates to", 4

19, " evaluates to", 11

20, " evaluates to", 18

21, " evaluates to", 25

22, " evaluates to", 1

23, " evaluates to", 8

24, " evaluates to", 15

25, " evaluates to", 22

26, " evaluates to", 29

27, " evaluates to", 5

28, " evaluates to", 12

29, " evaluates to", 19

30, " evaluates to", 26 **(20)**

Not a good idea if solving mod 1009! Maple has a modular solve command:

$msolve(7 \cdot x + 2 - 1, 31)$

$$\{x = 22\} \quad \textbf{(21)}$$

Diophantine equations - we want integer solutions so we would use isolve

$isolve(x^2 - n \cdot y^2 = 1)$

$$\{n = \_Z1^2 - 1, x = \_Z1, y = -1\} \quad \textbf{(22)}$$

Gaussian Integers - the GaussInt package deals with Gaussian integers:

$with(GaussInt);$

[*GIbasis*, *GIchrem*, *GIdivisor*, *GIfacpoly*, *GIfacset*, *GIfactor*, *GIfactors*, *GIgcd*, *GIgcdex*, *GIhermite*, *GIissqr*, *GIlcm*, *GImcmbine*, *GImod*, *GInearest*, *GInodiv*, *GInorm*, *GInormal*, *GIorder*, *GIphi*, *GIprime*, *GIquadres*, *GIquo*, *GIrem*, *GIroots*, *GIsieve*, *GIsmith*, *GIsqrfree*, *GIsqrt*, *GIunitnormal*] **(23)**

We can factor over the Gaussian Integers:

$GIfactor(10 + 10 \cdot I);$

$$(-\mathrm{I})\ (1 + \mathrm{I})^3\ (1 + 2\,\mathrm{I})\ (1 - 2\,\mathrm{I}) \quad \textbf{(24)}$$

We can take an approximate square root over the Gaussian Integers.

$GIsqrt(10 + 10 \cdot I);$

$$3 + \mathrm{I} \quad \textbf{(25)}$$

Complex Numbers

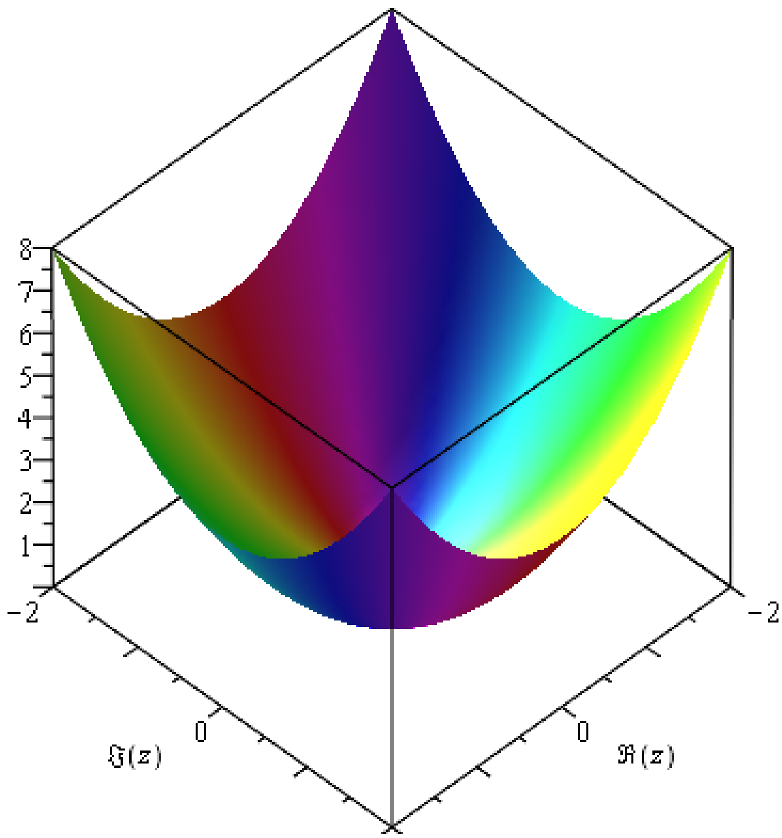
Plotting a complex number on an Argand diagram is easy with the Complex Numbers Tutor. We can also plot complex functions using complexplot and complexplot3d from the plots package:Error, missing operator or `;`

$with(plots):$

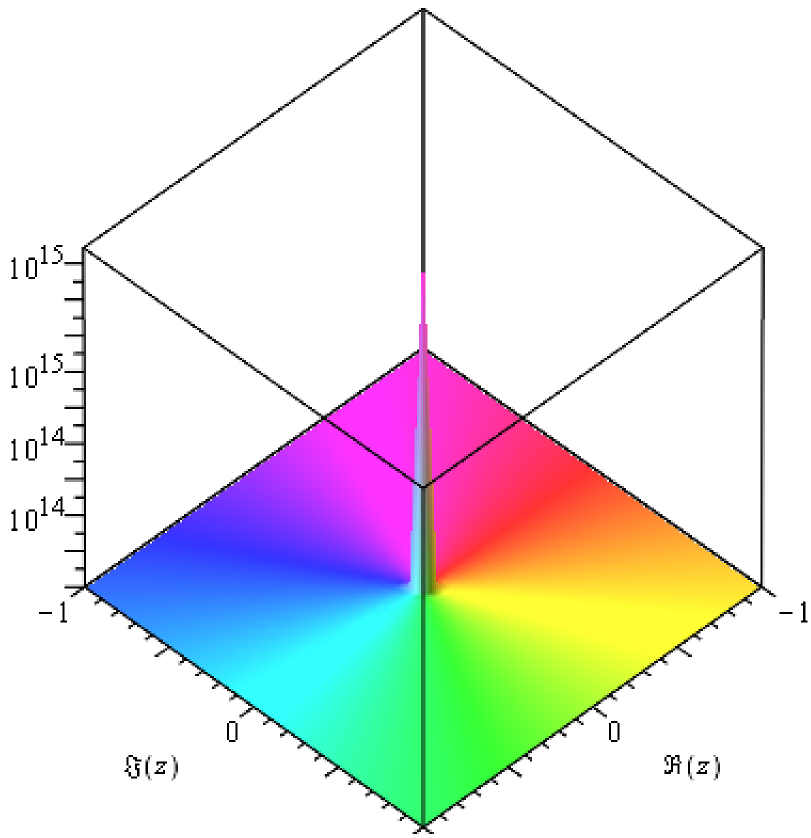
$f := z^2;$

$$z^2 \quad \textbf{(26)}$$

$complexplot3d(f, z = -2 - 2 \cdot I..2 + 2 \cdot I);$



$$complexplot3d\left(\frac{1}{z}, z=-1-1\cdot I..1+1\cdot I\right)$$



Wherever possible, Maple solves over the complex numbers:

$solve(x^2 + x + 1)$

$$-\frac{1}{2} + \frac{1}{2}\,\mathrm{I}\sqrt{3},\ -\frac{1}{2} - \frac{1}{2}\,\mathrm{I}\sqrt{3} \tag{27}$$

And it can deal with polynomials with complex coefficients:

$solve(\,(1+I)\cdot x^2 - I\cdot x + 1)$

$$\left(\frac{1}{4} - \frac{1}{4}\,\mathrm{I}\right)\left(\mathrm{I} + \sqrt{-5-4\,\mathrm{I}}\right),\ \left(-\frac{1}{4} + \frac{1}{4}\,\mathrm{I}\right)\left(-\mathrm{I} + \sqrt{-5-4\,\mathrm{I}}\right) \tag{28}$$

If we want to make sure we get all possible solutions (including complex ones) we use the allsolutions option:

$solve(\exp(z) = -1, z)$

$$\mathrm{I}\,\pi \tag{29}$$

$solve(\exp(z) = -1, z, allsolutions)$

$$\mathrm{I}\,\pi + 2\,\mathrm{I}\,\pi\,\_Z1\sim \tag{30}$$

Most functions work over the complexes as well as the reals. If we want to make sure we are evaluating over the complex numbers we can use evalc which converts a number into a+b*I:

$evalc\left(\left(\frac{1}{4} - \frac{1}{4}\,\mathrm{I}\right)\left(\mathrm{I} + \sqrt{-5-4\,\mathrm{I}}\right)\right)$

$$\frac{1}{8}\sqrt{-10+2\sqrt{41}}+\frac{1}{4}-\frac{1}{8}\sqrt{10+2\sqrt{41}}+\mathrm{I}\left(-\frac{1}{8}\sqrt{-10+2\sqrt{41}}+\frac{1}{4}-\frac{1}{8}\sqrt{10+2\sqrt{41}}\right) \quad \textbf{(31)}$$

Plus we can do normal complex number operations like taking the conjugate:

$$conjugate\left(\left(\frac{1}{4}-\frac{1}{4}\,\mathrm{I}\right)\left(\mathrm{I}+\sqrt{-5-4\,\mathrm{I}}\right)\right)$$

$$\left(\frac{1}{4}+\frac{1}{4}\,\mathrm{I}\right)\left(-\mathrm{I}+\sqrt{-5+4\,\mathrm{I}}\right) \quad \textbf{(32)}$$

Curves

We use the plots package to deal with plotting functions.

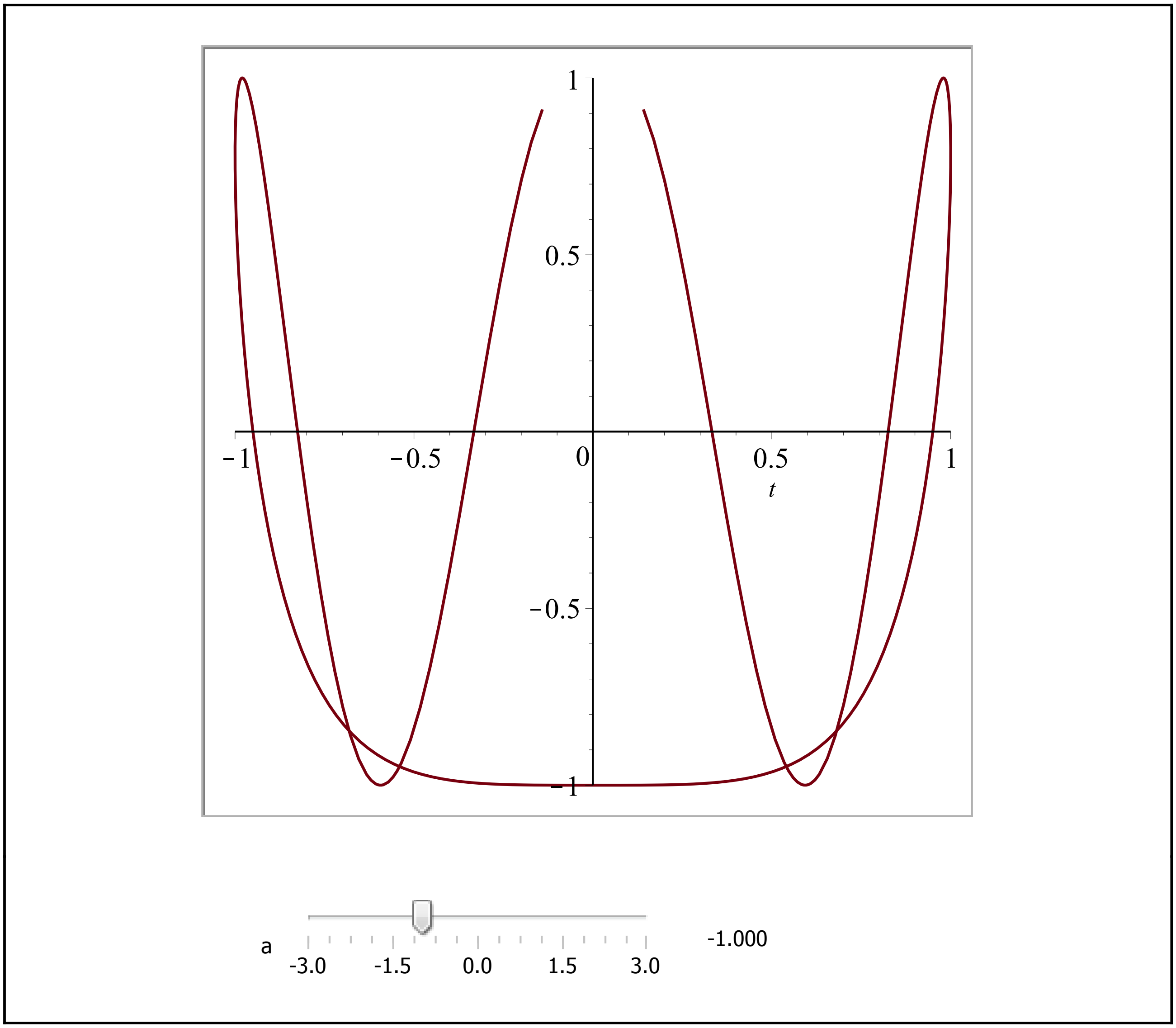
with(*plots*);

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(33)**

Often the Plot Builder assistant is more useful. It can automatically suggest how to plot things: like polar plots, parametric plots, and more.

We can also include optional parameters: [a*sin(a*t),cos(t^2)/a]

$$Explore\left(plot\left(\left[a\sin(t\,a),\ \frac{\cos(t^2)}{a},\ t=-3\,..3\right],\ labels=[t,\ ""]\right),\ 'parameters'=[a=-3.\,..3.],\ 'initialvalues'=[a=-1]\right)$$



Convert from one set of coordinates to another using symblic algebra with the changecoords command. But there is a problem! Plots has redefined the changecoords command to deal with plotting (it's really useful!) so we have to restart before we can use changecoords.

restart

$f := changecoords\left((x-y)^2, [x, y], \text{polar}, [r, \text{theta}] \right);$

$$r^2 \left(\cos(\theta) - \sin(\theta)\right)^2 \tag{34}$$

And back using a substitution - as we are substituting with an algebraic expression we use algsubs.

$algsubs(r \cdot \sin(\text{theta}) = y, algsubs(r \cdot \cos(\text{theta}) = x, expand(f)))$

$$x^2 - 2\,x\,y + y^2 \tag{35}$$

$factor(algsubs(r \cdot \sin(\text{theta}) = y, algsubs(r \cdot \cos(\text{theta}) = x, expand(f))))$

$$(x-y)^2 \tag{36}$$

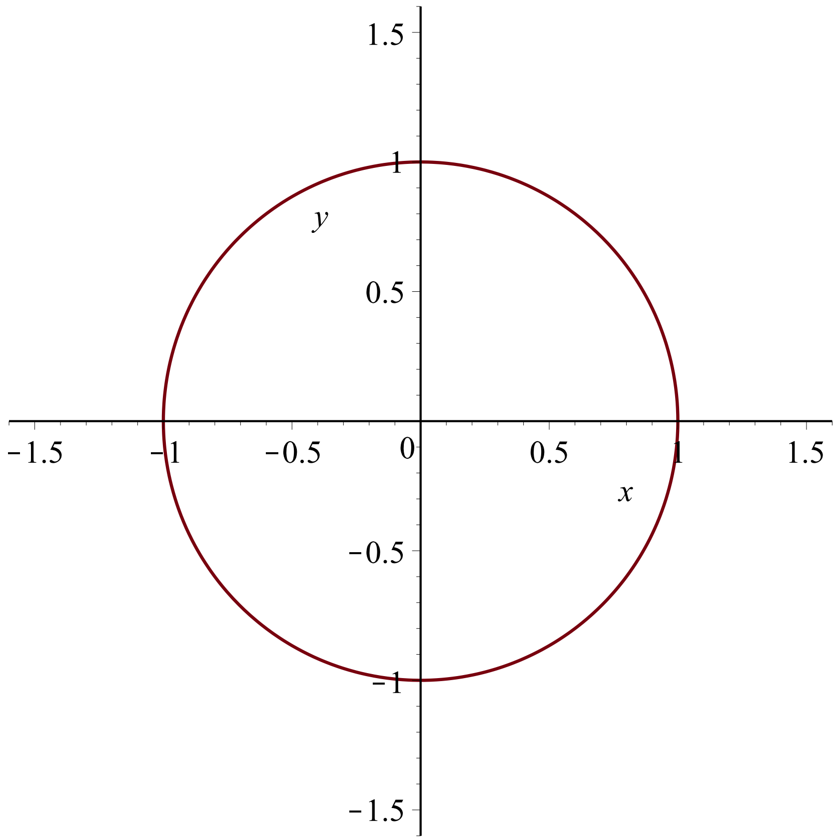
Differential Equations

Let's restart to make sure all our commands are okay.

restart

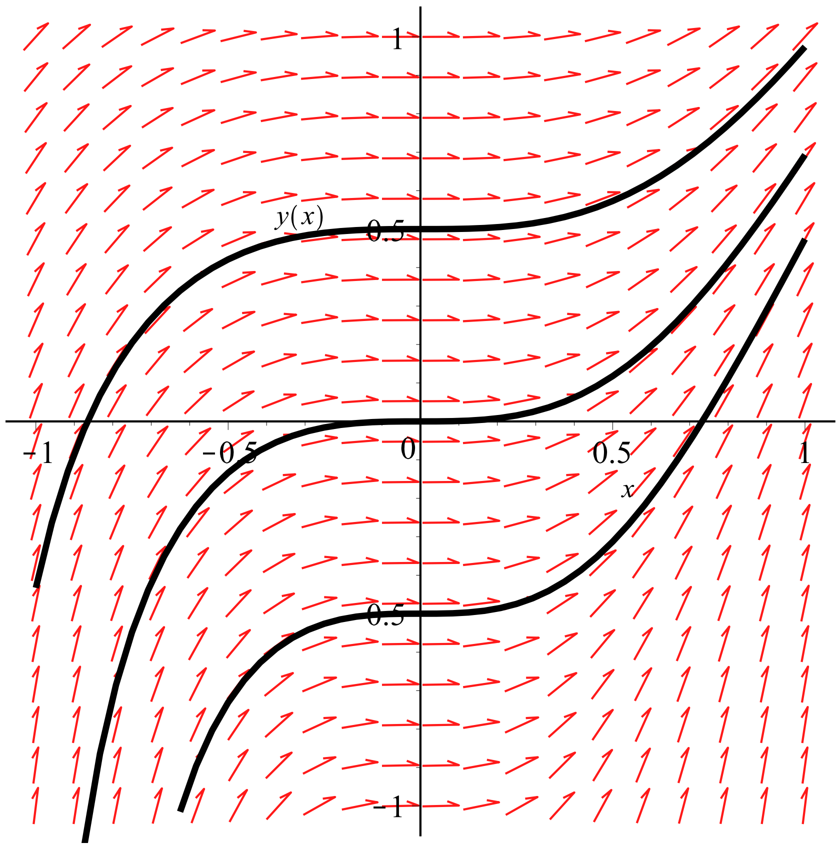
with(*DEtools*)

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*] **(37)**



We can plot some common Tangent fields using a Maple tutor, or make our own with DEplot

$DEplot\left(diff(y(x), x) - 3\,x^2 \exp(-y(x)), y(x), x = -1..1, y = -1..1, [[0, 0], [0, -0.5], [0, 0.5]], linecolor = black\right)$



Maple also has a really good assistant for Ordinary Differential Equations. But let's try a couple on our own:

Separation of variables - consider the equation we used above. It is separable.

$ode := diff(y(x), x) - 3\,x^2 \exp(-y(x));$

$$\frac{d}{dx}\, y(x) - 3\,x^2\,e^{-y(x)} \tag{38}$$

We can use Maple's inbuilt DE solver, but it doesn't show us how to find the solution!

$dsolve(ode)$

$$y(x) = \ln\left(x^3 + 3\,\_C1\right) \tag{39}$$

We can manually divide by the y-expression.

$dividedode := \dfrac{ode}{\exp(-y(x))};$

$$\frac{\frac{d}{dx}\, y(x) - 3\,x^2\,e^{-y(x)}}{e^{-y(x)}} \tag{40}$$

And then we can integrate with respect to x, making sure we simplify to get it in a readable form. Be careful - there is no constant of integration!

$simplify(int(dividedode, x))$

$$e^{y(x)} - x^3 \tag{41}$$

We can now solve for y - but this only gives us 1 solution, not them all.

$solve\left(e^{y(x)} - x^3 = 0, y(x)\right)$

$$3 \ln(x) \tag{42}$$

If we are using integrating Factors we can call DETools[intfactor]:

$ode := x \cdot diff(y(x), x) - 2 \cdot y(x) - x^4 \cdot \sin(x);\ odeadvisor(ode)$

$$x \left(\frac{d}{dx} y(x)\right) - 2\, y(x) - x^4 \sin(x)$$

$$[\_linear] \tag{43}$$

$intfactor(ode)$

$$\frac{1}{x^3} \tag{44}$$

Maple can even deal with non-homogeneous 2nd order ODEs: for example Damped oscillation

$ode := diff(y(x), x, x) + a \cdot diff(y(x), x) + w^2 \cdot y(x)$

$$\frac{d^2}{dx^2} y(x) + a \left(\frac{d}{dx} y(x)\right) + w^2\, y(x) \tag{45}$$

$odeadvisor(ode)$

$$[[\_2nd\_order, \_missing\_x]] \tag{46}$$

$dsolve(ode)$

$$y(x) = \_C1\, e^{\left(-\frac{1}{2} a + \frac{1}{2} \sqrt{a^2 - 4 w^2}\right) x} + \_C2\, e^{\left(-\frac{1}{2} a - \frac{1}{2} \sqrt{a^2 - 4 w^2}\right) x} \tag{47}$$

We see that the solution includes a squareroot - we should worry about what happens if a^2-4w^2 is negative so we use the 'assuming' option.

$dsolve(ode)$ assuming $a^2 - 4 \cdot w^2 > 0$

$$y(x) = \_C1\, e^{\frac{1}{2} \left(-a + \sqrt{a^2 - 4 w^2}\right) x} + \_C2\, e^{-\frac{1}{2} \left(a + \sqrt{a^2 - 4 w^2}\right) x} \tag{48}$$

$dsolve(ode)$ assuming $a^2 - 4 \cdot w^2 < 0$

$$y(x) = \_C1\, e^{-\frac{1}{2} a x} \sin\left(\frac{1}{2} \sqrt{-a^2 + 4 w^2}\, x\right) + \_C2\, e^{-\frac{1}{2} a x} \cos\left(\frac{1}{2} \sqrt{-a^2 + 4 w^2}\, x\right) \tag{49}$$

$specificode : subs(a = 1, w = 1, ode)$

$$\frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) \tag{50}$$